Exact and approximate solutions for marginalization with probabilistic population codes

Jeffrey M. Beck1,2, Wei Ji Ma1, Vidhya Navalpakkam, and Alexandre Pouget1
1University of Rochester, 2California Institute of Technology

In many tasks, human behavior has been shown to be nearly optimal in a Bayesian sense. This implies that neural computation is capable of optimally manipulating representations of probability distributions which represent both incoming evidence and prior knowledge. In particular, neurons must be able to perform two classes of Bayesian inference: evidence accumulation and marginalization (integration over hidden variables). Several theoretical studies have sought representations of probability distributions that could implement these inferences using neurons whose responses are proportional to probability or log probability. The main problem with these approaches is that they fail to account for the variability in neural responses, and in particular the Poisson-like variability that has been reported in cortex.

We show here that the Probabilistic Population Coding (PPC) framework [1] can provide an optimal solution to both evidence accumulation and marginalization, while accounting for neural variability. Specifically, we show that in the high-information limit, a linear PPC may be marginalized using only linear operations, a quadratic point non-linearity, and divisive normalization, operations that are believed to be used in real neural circuits. Additionally, we show that even when the large information assumption is not valid, the same set of neural operations can still leads to near-optimal levels of performance. This results from the fact that the PPC framework allows us to approximate the generative model when either (1) it induces no measurable information loss about the stimuli of interest or (2) there is insufficient observational data to distinguish between the approximate statistical model and the ground truth.

These results are illustrated in a neural implementation of a Bayesian model of a visual search task (See Poster #). The task is to detect and localize a target among distractors. Input patterns of activity obey statistics that correspond to a linear PPC and represent the values of a particular feature at each of several different spatial locations. The goal of the network is then to construct from the input pattern of activity, \( r_{\text{in}} \), a second population pattern of activity, \( r_{\text{out}} \), which also represents a linear PPC but for the probability that the target is present at each location. Since a line attractor can be used to optimally decode a linear PPC, we conclude that the operation \( r_{\text{out}} = f(r_{\text{in}}) \) along with the line attractor can be used to generate a saccade in the direction which maximizes the likelihood of fixating on the target or, alternatively, can generate a no-go signal when it is most likely that the target is not present. Thus, this three-layer network would generate an optimal sensorimotor transformation.

The Bayes-optimal computation for this task requires a series of operations which include exponentiation, and summation, and log transformation. Motivated by the structure of the optimal computation in the large-information limit, we show that a network which constructs \( r_{\text{out}} \) using only linear operations, a quadratic point non-linearity, and divisive normalization can be made to yield a linear PPC which performs both target detection and target localization at levels which are nearly indistinguishable from optimal. We also show that when \( r_{\text{out}} \) is constructed from either just linear combinations of \( r_{\text{in}} \) or just linear and quadratic terms but without divisive normalization, a significant degradation in performance results.

Reference