

# Recovery of Stimuli Encoded with Hodgkin-Huxley Neurons

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In the reverse correlation method [1] to neural encoding, the neural system is treated as a black box and both the input stimuli and output spike trains are assumed to be observable. Based on the correlation between the input and the output, the black box is identified as the kernel of a Wiener filter. When investigating problems arising in olfactory coding, an explicit characterization of odor stimuli is, however, not available. In addition, sensory systems are exposed to unknown stimuli in real-time. Under these conditions, investigations of sensory encoding of natural stimuli assume that only the spike train is observable and the neural system can be described as a non-linear dynamical system. The Hodgkin-Huxley neuron is the model of choice.

A non-linear perturbation analysis shows that the system of ordinary differential equations describing the Hodgkin-Huxley neuron with a weak stimulus accepts a solution consisting of a phase shift term and a small perturbation term. We derive a recovery algorithm that extracts information about the stimulus contained in the phase shift term and is embedded in the perturbation term. Results are obtained for both Hodgkin-Huxley neurons with deterministic and stochastic gating variables, respectively.

We show that a Hodgkin-Huxley neuron with deterministic gating variables is I/O equivalent with a project-integrate-and-fire (PIF) neuron with a variable threshold sequence. The PIF neuron integrates a projection of the stimulus onto the phase response curve that is, in turn, modulated by a phase shift process. The phase shift process is described by a differential equation that is stimulus driven. The PIF neuron generates a spike whenever a threshold value is achieved; the values of the threshold sequence are explicitly given. In the absence of the small perturbation term, the stimulus is tangentially coupled into the limit cycle of the Hodgkin-Huxley neuron and the PIF neuron reduces to an integrate-and-fire neuron [2], [3]. Building on the I/O equivalent PIF neuron, we provide a stimulus recovery algorithm and evaluate its performance.

We extend these results to noisy Hodgkin-Huxley neurons. This calls for generalizing the phase response curve to the case where the gating variables are modeled as stochastic processes. We provide a stochastic differential equation that describes the phase shift process and reduce the noisy Hodgkin-Huxley neuron to a stochastic analog of the PIF neuron obtained in the deterministic case. Finally, we provide an algorithm for stimulus recovery based on observing the spike train generated by a noisy Hodgkin-Huxley neuron.

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## References

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