Population Codes: Decoding Quadratic Forms

Charles H. Anderson[1]

[1]Washington University School of Medicine, St. Louis MO

We have argued for some time [1, 2] that the computation of quadratic forms by neuronal circuits is essential for contextual inference, dynamic routing of information, and dynamic control of motor outputs. In [2] it is shown that when a vector space $\mathbf{X}$ is represented by a large population of neurons with a broad range of monotonic tuning curves $\mathbf{a}(\mathbf{X})$, then the principal components of the tuning curves form a set of relatively smooth functions of the form

$$\mathbf{F}(\mathbf{X}) = \mathcal{R}\mathbf{a}(\mathbf{X}) = A_0 + \mathbf{A}_1 \cdot \mathbf{X} + \mathbf{X}^*\mathbf{A}_2\mathbf{X} + \ldots, \tag{1}$$

where the rotation matrix $\mathcal{R}$ diagonalizes the correlation matrix $C =< \mathbf{a}^*(\mathbf{X})\mathbf{a}(\mathbf{X}) >_{\mathbf{X}}$. This provides a mechanism by which neuronal circuits can compute quadratic forms. Several important facts about this observation not previously noticed are: 1) These functional forms constitute the primary information that can be extracted from the neuronal population by linear projections into the soma currents of other neurons; 2) the noise level of these functions is the same as the individual neuronal noise level, but is Gaussian regardless of the statistics of the neuronal noise because of the law of large numbers. (This makes the issue of Poisson statistics moot in redundant population code modeling); 3) recent fits to spatio-temporal receptive field properties utilize this form [3], providing neurobiological experimental support for the high level analysis leading to equation (1). In essence, these observations enable high level modeling to be combined with low level neuronal analysis (see [4]). For example, making the low dimensional system variables explicit should make it easier to extend the MT model by [5] to other systems. Also, point (1) implies it should be more efficient for those who want to model large neuronal circuits without modeling all the details of the individual neurons to use quadratic forms for their basic units rather then pseudo-neuron like units[6].

This work details how properties of the principle components $\mathbf{F}(\mathbf{X})$, such as the SNR, scale with neuronal number, the dimensionality of the space, and the distribution of the tuning curves. The efficiency of the representation drops as the dimensionality increases, which implies neurons should only respond to multiple parameters that interact with one another in the computation the circuit is performing. Also, for large dimensional spaces the neuronal background firing rates should be low, or zero, like many cortical neurons .

Acknowledgments
This work was supported by the Mathers Foundation and the McDonnell Center for Higher Brain Function.

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