A mathematical constant in the design of the visual cortex

Matthias Kaschube[1,2], Michael Schnabel[2], Siegrid Loewel[3], David M. Coppola[4], Leonard E. White[5] & Fred Wolf[2]

[1]Lewis-Sigler Institute for Integrative Genomics, Princeton University, Princeton NJ, USA, [2]MPI for Dynamics and Self-Organization and Bernstein-Center for Computational Neuroscience, Goettingen, Germany, [3]Institute of General Zoology and Animal Physiology, University Jena, Jena, Germany, [4]Department of Biology, Randolph-Macon College, Ashland, VA, USA, [5]Department of Community & Family Medicine, and Department of Neurobiology, Duke University Medical Center, Durham, NC, USA.

More than hundreds of tera byte of information would be necessary to describe the synaptic organization of the mammalian cerebral cortex. It is thus hardly conceivable that the roughly 20 thousand genes of a typical mammalian genome might be sufficient to directly specify the cerebral cortical structure. It has therefore been hypothesized that dynamical self-organization might play an important role in shaping neuronal circuits in the cerebral cortex. The notion of self-organization is meant to stress that in such systems although the process of structure formation is externally driven by e.g. sensory input, the structures that emerge are determined through interactions within the system it-self. Mostly, from the study of non-living systems, it is well understood that self-organization often exhibits universality, meaning that the structures formed are largely insensitive to the precise details of the underlying intrinsic interactions, and therefore common to many systems which potentially exhibit mayor differences in their detailed nature.

Here we demonstrate in three species separated in evolution by more than 50 million years that the layout of orientation columns in the visual cortex adheres to a set of universal quantitative laws. Most suggestive of a mathematical structure underlying this universality, the average number of pinwheel centers per orientation hyper-column in all species is virtually identical and statistically indistinguishable from the mathematical constant π. Here, a hyper-column is the natural area unit Λ^2, where Λ is the periodicity of the map. Universal behavior is also found for quantities characterizing the map design from scale of individual hyper-columns to the layout of the entire primary visual cortex. These results are obtained with a novel analysis method devised to selectively eliminate the many spurious pinwheels typically induced by high frequency noise and to account for spatial inhomogeneities often present in the structure of orientation maps.

To interprete these findings, we present a model for self-organization of visual cortical architecture that accounts quantitatively for all universal layout properties by a process of dynamical pattern selection. The model is derived from symmetry arguments and includes only key features of the visual cortex including non-local interactions mediated e.g. by the influence of long-range horizontal connections. The analysis of the model indicates that the development of orientation maps with the observed universal properties depends critically on assuming sufficient strength and range of non-local interactions (larger than at least 1Λ). In fact, in the limit of large interaction range, the ensemble averaged pinwheel density converges to π as shown analytically by perturbation theory.

Corroborating the idea of robust development through self-organization, we observe that the orientation map adheres to the universal behavior even if its organization exhibits marked overall inhomogeneities and when animals are raised under severe visual deprivation. Thus, because of the phenomenon of universality, self-organization appears suited to enable nervous systems to form similar neural architectures irrespective of differences in genetic background and sensory experience.